

Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours

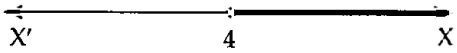
Maximum Marks: 80

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A and B are symmetric matrices, then ABA is: [1]
 - a) diagonal matrix
 - b) symmetric matrix
 - c) skew-symmetric matrix
 - d) scalar matrix
2. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as: [1]
 - a) $n_1 + n_2 - 2$
 - b) $n_1 + n_2 - 1$
 - c) $n_1 - n_2 + 2$
 - d) $n_1 - n_2 - 2$
3. Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is: [1]
 - a) ₹ 900
 - b) ₹ 800
 - c) ₹ 100
 - d) ₹ 700
4. The objective function of an LPP is [1]
 - a) a constrain
 - b) a relation between the variables
 - c) a function to be optimized
 - d) a function to be not optimized

5. The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for [1]
- a) $\lambda = 0$ b) $\lambda = 1$
c) $\lambda \in \mathbb{R} - \{1\}$ d) $\lambda = -1$
6. A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds, he must fire in order to have more than 50% chance of hitting it at least once is [1]
- a) 5 b) 7
c) 9 d) 11
7. There are 50 telephone lines in an exchange. The probability that any one of them will be busy is 0.1. The probability that all the lines are busy is [1]
- a) $\frac{5^0 e^{-5}}{0!}$ b) $1 - \frac{5^0 e^{-5}}{0!}$
c) $\frac{5^{50} e^{-5}}{50!}$ d) $1 - \frac{5^{50} e^{-5}}{50!}$
8. The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^5 + 3xy\left(\frac{d^3y}{dx^3}\right)^2 + y^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$, respectively, are [1]
- a) 2, 3 b) 1, 5
c) 3, 5 d) 3, 2
9. An outlet pipe can empty a cistern in 3 hours. The time taken by it to empty $\frac{3}{2}$ rd of the cistern is [1]
- a) 2 hours b) 6 hours
c) 4 hours d) 3 hours
10. In a 100 m race A and B are two participants. If A runs at 5 kilometer per hour and A gives B a start of 8 m and still beats him by 8 seconds, then the speed of B is: [1]
- a) 5.15 km/hr b) 4.4 km/hr
c) 4.14 km/hr d) 4.25 km/hr
11. A milkman mixed some water with milk to gain 25% by selling the mixture at the cost price. The ratio of water and milk respectively is: [1]
- a) 1 : 5 b) 5 : 4
c) 1 : 4 d) 4 : 5
12. The solution of the linear inequality in x represented on number line as [1]
- 
- a) $x \in [4, \infty)$ b) $x \in (4, \infty)$
c) $x \in (\infty, 4]$ d) $x \in (-\infty, 4)$
13. A cistern is filled in 20 minutes by three pipes A, B and C. The pipe C is twice as fast as B and pipe B is thrice as fast as pipe A. How much time will pipe A alone take to fill the tank? [1]
- a) 180 minutes b) 205 minutes
c) 200 minutes d) 352 minutes
14. The point in the half-plane $2x + 3y - 12 \geq 0$ is: [1]



- a) (7, 8) b) (-7, -8)
c) (7, -8) d) (-7, 8)
15. $\int e^x \{f(x) + f'(x)\} dx =$ [1]
a) $e^x - f(x) + C$ b) $e^x f(x) + C$
c) $2e^x f(x) + C$ d) $e^x + f(x) + C$
16. If we reject the null hypothesis, we might be making [1]
a) A correct decision b) A wrong decision
c) Type-II error d) Type-I error
17. $\int \frac{x^3}{x+1} dx$ is equal to [1]
a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log |1+x| + C$ b) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log |1-x| + C$
c) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log |1-x| + C$ d) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log |1+x| + C$
18. Which of the following can't be a component for a time series plot? [1]
a) Noise b) Trend
c) None of these d) Seasonality
19. **Assertion (A):** If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k such that $A^2 = kA - 2I$, is -1. [1]
Reason (R): If A and B are square matrices of same order, then $(A+B)(A+B)$ is equal to $A^2 + AB + BA + B^2$.
a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** If $y = \sin x$, then $\frac{d^3 y}{dx^3} = -1$ at $x = 0$. [1]
Reason (R): If $y = f(x) \cdot g(x)$, then $\frac{dy}{dx} = f(x) \cdot \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$.
a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

Section B

21. Assuming a four yearly cycle, calculate the trend by the method of moving averages from the following data: [2]

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Value	12	25	39	54	70	87	105	100	82	65

22. Calculate the compound interest on a sum of ₹ 2,50,000 at 8% p.a for 2 years compounded half yearly. [2]

[Use $(1.04)^4 = 1.169$]

OR

Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

23. Evaluate: $\int_0^1 \frac{xe^x}{(x+1)^2} dx$ [2]

[2]



24. Can you find the values of x and y so that the matrices $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ may be equal?
OR

If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify that $(A + B)' = A' + B'$

25. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement. [Use $(0.9)^5 = 0.59049$] [2]

Section C

26. Solve the differential equation: $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$ [3]
OR

In a bank principal increases at the rate of 5% per year. In how many years ₹ 1000 double itself.

27. A firm anticipates an expenditure of ₹ 50,000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of year into a sinking fund earning interest 5% per annum. [Given $\log 1.05 = 0.0212$, $\text{antilog } (0.2120) = 1.629$] [3]

28. A tyre manufacturer estimates that x (thousand) radial tyres will be purchased i.e. demanded by wholesalers when price is $p = D(x) = 90 - \frac{x^2}{10}$ thousand rupees per tyre and the same number of tyres will be supplied when the price is $p = S(x) = \frac{1}{5}x^2 + x + 50$ thousand rupee per tyre. [3]

i. Find the equilibrium price and the quantity supplied and demanded at that price.

ii. Determine the consumer's and producer's surplus at the equilibrium price.

29. If X is a normal variate with mean 30 and S.D. 5. Find: [3]

i. $P(26 \leq X \leq 40)$

ii. $P(X \geq 45)$

iii. $P(|X - 30| > 5)$

OR

A factory produces bulbs. The probability that one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

i. none of the bulbs is defective

ii. exactly two bulbs are defective

iii. more than 8 bulbs work properly.

30. The following figures relate to the profits of a commercial concern for 8 years. [3]

Years	2016	2017	2018	2019	2020	2021	2022	2023
Profit (₹)	15,420	15,470	15,520	21,020	26,500	31,950	35,600	34,900

Find the trend of profits by the method of three-yearly moving averages.

31. A soap manufacturing company was distributing a particular brand of a soap through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop was 140 dozens, and after the campaign, a sample of 26 shops was taken and mean sales was found to be 147 dozens with standard deviation 16. [3]

Can you consider the advertisement effective?

(Use $t_{25}(0.05) = 2.06$)

Section D

32. A factory manufactures two types of screws, A and B, each type requiring the use of two machines-an automatic [5]



and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machine to manufacture a package of screws **A**, while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a package of screws **B**. Each machine is available for atmost 4 hours on any day. The manufacturer can sell a package of screws **A** at a profit of ₹ 7 and of screws **B** at a profit of ₹10. Assuming that he can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

OR

A brick manufacturer has two depots, A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in ₹ of transporting 1000 bricks to the builders from the depots are given below:

From/To	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer fulfill the orders so as to keep the cost of transportation minimum? Formulate the above linear programming problem.

33. Evaluate: $7^{258} \pmod{13}$ [5]
34. After corresponding 100 pages of a book, the proofreader finds that there are, on average, 4 errors in 10 pages. [5]
How many pages would one expect to find with 0, 1, 2, 3 and 4 errors in 1000 pages of the first print of the book? (Use $e^{-0.4} = 0.6703$)

OR

A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation? Also, calculate the probability of 3 or less overflow floods in a 10 years interval.

[Given $e^{-2} = 0.13534$]

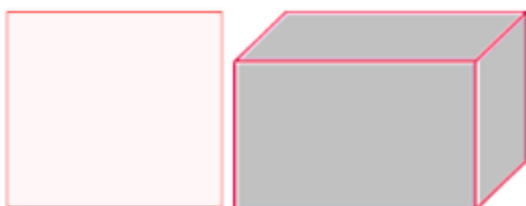
35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the [5]
formula, calculate CAGR of Vikas's investment given below:
Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1.4)^{1/6} = 1.058$]

Section E

36. **Read the text carefully and answer the questions:** [4]
Yash wants to prepare a handmade gift box for his friend's birthday at his home. For making lower part of the box, he took a square piece of paper of each side equal to 10 cm.



- (a) If x cm be the size of the square piece cut from each corner of the paper of size 10cm, then a possible

value of x will be given by interval:

- (b) Volume of the open box formed by folding up the cutting corner can be expressed as:
- (c) Find the value of x for which $\frac{dV}{dx} = 0$?

OR

Yash is interested to maximise the volume of the box, So what will be the side of the square to be cut to maximise the volume?

37. Read the text carefully and answer the questions:

[4]

Understanding Perpetuity

An annuity is a stream of cash flows. A perpetuity is a type of annuity that lasts forever, into perpetuity. The stream of cash flows continues for an infinite amount of time. In finance, a person uses the perpetuity calculation in valuation methodologies to find the present value of a company's cash flows when discounted back at a certain rate.

An example of a financial instrument with perpetual cash flows was the British-issued bonds known as consols, which the Bank of England phased out in 2015. By purchasing a consol from the British government, the bondholder was entitled to receive annual interest payments forever.

Perpetuity Present Value Formula

The formula to calculate the present value of perpetuity or security with perpetual cash flows is as follows:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} \dots = \frac{C}{r}$$

where:

PV present value

C = cash flow

r = discount rate

- (a) Find the present value of a perpetuity of ₹ 900 payable at the end of each year, if money is worth 5% per annum.
- (b) Find the present value of a perpetuity of ₹ 500 payable at the end of each quarter, if money is worth 8% per annum.
- (c) Find the present value of a perpetuity of ₹ 300 payable at the beginning of every 6 months, if money is worth 6% per annum.

OR

What amount is received at the end of every 6 months forever, if ₹ 72000 kept in a bank earns 8% per annum compounded half yearly?

38. Write the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

[4]

OR

The total sales (S) in thousands of rupees of a firm selling two products X and Y is given by the relationship: $S = a + bX + cY$. Sales data from January-March are given below:

Month	X	Y	Total Sales (S)
January	2	3	12
February	6	2	13
March	5	3	15

Using determinant method, determine the sales in the next month when it sells 4 units of X and 5 units of Y.



Solution

Section A

1.
(b) symmetric matrix
Explanation:
 $A' = A$ & $B' = B$
 $(ABA)' = A' (AB)'$
 $= A'B'A'$
 $= ABA$
Therefore ABA is symmetric matrix
2. (a) $n_1 + n_2 - 2$
Explanation:
 $n_1 + n_2 - 2$
3.
(b) ₹ 800
Explanation:
₹ 800
4.
(c) a function to be optimized
Explanation:
A Linear programming problem is a linear function (also known as an objective function) subjected to certain constraints for which we need to find an optimal solution (i.e. either a maximum/minimum value) depending on the requirement of the problem.
From the above definition, we can clearly say that the Linear programming problem's objective is to either maximize/minimize a given objective function, which means to optimize a function to get an optimum solution.
5.
(b) $\lambda = 1$
Explanation:
$$\begin{vmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{vmatrix} = 0$$
$$\Rightarrow 2(0 - 7) + 1(4\lambda + 7) + 3(\lambda - 0) = 0$$
$$\Rightarrow -14 + 4\lambda + 7 + 3\lambda = 0$$
$$\Rightarrow 7\lambda - 7 = 0 \Rightarrow \lambda = 1$$
6.
(b) 7
Explanation:
Given $p = \frac{1}{10} \Rightarrow q = \frac{9}{10}$
Let n be the number of rounds
 $P(x \geq 1) = 1 - P(x = 0)$
 $\Rightarrow P(x \geq 1) \geq 0.5$
 $\Rightarrow 1 - P(X = 0) \geq 0.5$
 $\Rightarrow P(X = 0) \leq 0.5$

$$\Rightarrow 0.9n \leq 0.5$$

Using log table,

$$n \leq 6.572 \approx 7$$

He must fire in order to have more than 50% chance of hitting the target at least once.

7.

(c) $\frac{5^{50}e^{-5}}{50!}$

Explanation:

Given $n = 50$, $P = 0.1$, so $\lambda = np = 50 \times 0.1 = 5$

So, $P(X = 50) = \frac{5^{50} \cdot e^{-5}}{50!}$

8.

(d) 3, 2

Explanation:

The highest order derivative present is $\frac{d^3y}{dx^3}$. So, its order is 3 and its exponent is 2, so its degree is 2.

9.

(a) 2 hours

Explanation:

The outlet pipe empties the one complete cistern in 3 hours

Time taken to empty $\frac{2}{3}$ Part of the cistern

$$= \frac{2}{3} \times 3$$

$$= 2 \text{ hours}$$

10.

(c) 4.14 km/hr

Explanation:

$$\text{A's Speed} = \frac{\text{Distance}}{\text{Time Travelled}}$$

$$\Rightarrow \text{A's Speed} = 5 \text{ kmph} = \frac{100 \text{ m}}{\text{Time Travelled}}$$

$$\Rightarrow \text{Total time taken by A to complete 100m} = \frac{100}{\left(\frac{5 \times 1000}{3600}\right)} \text{ seconds} = 72 \text{ seconds}$$

$$\Rightarrow \text{B's Speed} = \frac{\text{Distance Travelled by B}}{\text{Time T aken by B}} = \frac{\frac{(100-8)}{1000}}{\frac{(7+8)}{3600}} \text{ kmph} = \frac{92 \times 36}{800} \text{ kmph} = 4.14 \text{ kmph}$$

11.

(c) 1 : 4

Explanation:

Cost price of 1 litres of milk = ₹ 100

$\therefore$ Mixture sold for ₹ 125

$$= \frac{125}{100} = \frac{5}{4} \text{ litres}$$

$$\therefore \text{Quantity of mixture} = \frac{5}{4} \text{ litres}$$

$$\therefore \text{Quantity of milk} = 1 \text{ litre}$$

$$\therefore \text{Quantity of water} = \frac{5}{4} - 1 = \frac{1}{4} \text{ litre}$$

$$\therefore \text{Required ratio} = \frac{1}{4} : 1 = 1 : 4$$

12.

(b) $x \in (4, \infty)$

Explanation:

$$x \in (4, \infty)$$

13.

(c) 200 minutes

Explanation:

Suppose pipe A alone takes x hours to fill the tank.

Then pipes B and C will take $\frac{x}{3}$ and $\frac{x}{6}$ hours respectively to fill the tank.

$$\therefore \frac{1}{x} + \frac{3}{x} + \frac{6}{x} = \frac{1}{20}$$

$$\Rightarrow \frac{10}{x} = \frac{1}{20}$$

$$\Rightarrow x = 200 \text{ hours}$$

$\therefore$ pipe A alone will take 200 hours to fill the tank.

14. (a) (7, 8)

Explanation:

(7, 8)

15.

(b) $e^x f(x) + C$

Explanation:

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$t = e^x f(x)$$

$$\frac{dt}{dx} = e^x \cdot \frac{d}{dx}(f(x)) + f(x) \frac{d}{dx}(e^x)$$

$$= e^x f'(x) + f(x) \cdot e^x$$

$$dt = e^x (f'(x) + f(x)) dx$$

$$\int e^x \{f(x) + f'(x)\} dx = \int dt = t + C$$

$$= e^x f(x) + C$$

16.

(d) Type-I error

Explanation:

Type-I error

17. (a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log |1 + x| + C$

Explanation:

$$\text{Given: } \int \frac{x^3}{x+1} dx$$

$$\Rightarrow \frac{x^3}{x+1} = \frac{x^3+1-1}{x+1}$$

$$\Rightarrow \frac{x^3+1}{x+1} - \frac{1}{x+1} = \frac{(x+1)(x^2-x+1)}{x+1} - \frac{1}{x+1}$$

$$\Rightarrow \int \frac{x^3}{x+1} dx = \int \left((x^2 - x + 1) - \frac{1}{x+1} \right) dx$$

$$\Rightarrow \int (x^2 - x + 1) dx - \int \frac{1}{x+1} dx = \frac{x^3}{3} - \frac{x^2}{2} + x - \ln |1 + x| + C$$

18.

(c) None of these

Explanation:

None of these

19.

(d) A is false but R is true.

Explanation:

Assertion: Given, $A^2 = kA - 2I$

$$\Rightarrow AA = kA - 2I$$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By definition of equality of matrix, the given matrices are equal and their corresponding elements are equal.

Now, comparing the corresponding elements, we get

$$3k - 2 = 1 \Rightarrow k = 1$$

$$\Rightarrow -2k = -2 = k = 1$$

$$\Rightarrow 4k = 4 \Rightarrow k = 1$$

$$\Rightarrow -4 = -2k - 2 \Rightarrow k = 1$$

Hence, $k = 1$

Reason: We have,

$$(A + B)(A + B) = A(A + B) + B(A + B)$$

$$= A^2 + AB + BA + B^2$$

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

Section B

Calculation of 4-year centred moving average:

Year	Value	4-yearly moving total	4-yearly moving average	4-yearly centre moving average
1984	12			
1985	25	130	32.5	
1986	39			39.75
		188	47	
1987	54			54.75
		250	62.5	
1988	70			70.75
		316	79	
1989	87			84.75
		362	90.5	
1990	105			92
		374	93.5	
1991	100			90.75
		352	88	
1992	82			
1993	65			

21.

22. $P = ₹ 2,50,000$, $r = 8\%$ p.a. = 4% half yearly

$n = 2$ years = 4 half years

$$\therefore \text{C.I.} = 25000 \left[\left(1 + \frac{4}{100} \right)^4 - 1 \right]$$

$$= 250000[(1.04)^4 - 1]$$

$$= 250000(1.169 - 1)$$

$$= 250000 \times 0.169$$

$$= ₹ 42,250$$

$P = ₹ 2,50,000$, $r = 8\%$ p.a. = 4% half yearly

$n = 2$ years = 4 half years

$$\therefore \text{C.I.} = 25000 \left[\left(1 + \frac{4}{100} \right)^4 - 1 \right]$$

$$= 250000[(1.04)^4 - 1]$$

$$= 250000(1.169 - 1)$$

$$= 250000 \times 0.169$$

$$= ₹ 42,250$$

OR

We have,

$$r = \frac{10}{100} = 0.1$$

$$m = 12$$

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.1}{12}\right)^{12} - 1$$

$$= (1.00833)^{12} - 1$$

$$= 1.1047 - 1$$

$$= 0.1047$$

Thus, the effective rate of interest is 10.47%, which means that the rate of 10.47% compounded annually yield the same interest as the nominal rate 10% compounded monthly.

23. Let $I = \int_0^1 \frac{xe^x}{(1+x)^2} dx$

$$\Rightarrow I = \int_0^1 e^x \left(\frac{x+1-1}{(1+x)^2} \right) dx$$

$$\Rightarrow I = \int_0^1 e^x \left(\frac{(x+1)-1}{(1+x)^2} \right) dx$$

$$\Rightarrow I = \int_0^1 e^x \left(\frac{(x+1)}{(1+x)^2} - \frac{1}{(1+x)^2} \right) dx$$

$$\Rightarrow I = \int_0^1 e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right) dx$$

Put $f(x) = \frac{1}{1+x}$

$$f'(x) = -\frac{1}{(1+x)^2}$$

We know that,

$$\int e^x [f(x) - f'(x)] dx = e^x f(x) + c$$

$$\Rightarrow I = e^1 \left(\frac{1}{1+1} \right) - e^0 \left(\frac{1}{1+0} \right)$$

24. $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$$\Rightarrow 3x+7=0, 5=y-2, y+1=8 \text{ and } 2-3x=4$$

$$\Rightarrow x = -\frac{7}{3}, y = 7, y = 7 \text{ and } x = -\frac{2}{3}.$$

We note that $x = -\frac{7}{3}$ and $x = -\frac{2}{3}$ cannot be true simultaneously, therefore, the given matrices cannot be equal.

OR

$$A + B = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\Rightarrow (A+B)' = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\Rightarrow (A+B)' = A' + B'$$

25. Juice contained in the container after final replacement

$$= 50 \left(1 - \frac{5}{50}\right)^5 = 50 \left(\frac{9}{10}\right)^5$$

$$= 50 \times 0.59049 = 29.5 \text{ litres}$$

Section C

26. The given differential equation is

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2} \dots (i)$$

This is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{x \log x} \text{ and } Q = \frac{2}{x^2}$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{1}{t} dt}, \text{ where } t = \log x$$

$$\Rightarrow \text{I.F.} = e^{\log t} = t = \log x$$

Multiplying both sides of (i) by I.F. = log x, we get

$$\log x \frac{dy}{dx} + \frac{1}{x}y = \frac{2}{x^2} \log x$$

Integrating both sides with respect to x, we get

$$y \log x = \int \frac{2}{x^2} \log x \, dx + C \text{ [Using: } y(\text{I.F.}) = \int Q(\text{I.F.}) \, dx + c]$$

$$\Rightarrow y \log x = 2 \int \log x \, x^{-2} dx + C$$

$$\Rightarrow y \log x = 2 \left\{ \log x \left(\frac{x^{-1}}{-1} \right) - \int \frac{1}{x} \left(\frac{x^{-1}}{-1} \right) dx \right\} + C$$

$$\Rightarrow y \log x = 2 \left\{ -\frac{\log x}{x} + \int x^{-2} dx \right\} + C$$

$$\Rightarrow y \log x = 2 \left\{ -\frac{\log x}{x} - \frac{1}{x} \right\} + C$$

$$\Rightarrow y \log x = -\frac{2}{x} (1 + \log x) + C, \text{ which gives the required solution.}$$

OR

Let P be the principal at any time t. Then,

$$\frac{dP}{dt} = \frac{5P}{100}$$

$$\Rightarrow \frac{dP}{dt} = \frac{P}{20}$$

$$\Rightarrow \frac{1}{P} dP = \frac{1}{20} dt$$

Integrating both sides, we get

$$\int \frac{1}{P} dP = \int \frac{1}{20} dt$$

$$\Rightarrow \log P = \frac{1}{20} t + \log C$$

$$\Rightarrow \log \frac{P}{C} = \frac{1}{20} t$$

$$\Rightarrow P = C e^{\frac{t}{20}} \dots (i)$$

It is given that P = 1000 when t = 0.

Substituting these values in (i), we get

$$1000 = C$$

Substituting C = 1000 in (i), we get

$$P = 1000 e^{\frac{t}{20}} \dots (ii)$$

Let t_1 years be the time required to double the principal i.e. at $t = t_1$, $P = 2000$.

Substituting these values in (ii), we get

$$2000 = 1000 e^{\frac{t_1}{20}}$$

$$\Rightarrow e^{\frac{t_1}{20}} = 2 \Rightarrow \frac{t_1}{20} = \log_e 2 \Rightarrow t_1 = 20 \log_e 2$$

Hence, the principal doubles in $20 \log_e 2$ years.

27. Given, A = ₹ 5,00,000, r = 5% and n = 10

$$\text{Using formula, } A = p \left[\frac{(1+i)^n - 1}{i} \right]$$

$$\text{where } i = \frac{r}{100}$$

$$\Rightarrow 500000 = p \left[\frac{(1+0.05)^{10} - 1}{0.05} \right]$$

$$p = \frac{500000 \times 0.05}{(1.05)^{10} - 1}$$

$$\text{Now, let } x = (1.05)^{10}$$

Taking log both sides, we get

$$\log x = 10 \log (1.05)$$

$$= 10 \times 0.0212$$

$$= 0.2120$$

$$\Rightarrow x = \text{antilog } (0.2120)$$

$$= 1.629$$

$$\text{Thus, } (1.05)^{10} = 1.629$$

$$\text{Now, } p = \frac{500000 \times 0.05}{1.629 - 1}$$

$$= \frac{25000}{0.629}$$

$$= 39745.63$$

Hence, the company should deposit X 39745.63 every year into the sinking fund.

28. The equilibrium point (x_0, p_0) is the point at which the demand and supply curves intersect. Therefore, the equilibrium point is obtained by setting $D(x) = S(x)$.

Now, $D(x) = S(x)$

$$\Rightarrow 90 - \frac{x^2}{10} = \frac{x^2}{5} + x + 50$$

$$\Rightarrow \frac{3}{10}x^2 + x - 40 = 0 \Rightarrow 3x^2 + 10x - 400 = 0 \Rightarrow (x - 10)(3x + 40) = 0 \Rightarrow x - 10 = 0 \Rightarrow x = 10$$

Putting $x = 10$ in $p = D(x)$ or $p = S(x)$, we obtain $p = 80$

Thus, the equilibrium point is $(x_0, p_0) = (10, 80)$ i.e. the equilibrium occurs at ₹ 80,000 per tyre, when 10,000 tyres are supplied and demanded.

i. We have, $x_0 = 10$ and $p_0 = 80$. The consumer's surplus (CS) is given by

$$\begin{aligned} CS &= \int_0^{x_0} D(x)dx - p_0 x_0 \\ \Rightarrow CS &= \int_0^{10} \left(90 - \frac{x^2}{10}\right) dx - 80 \times 10 = \left[90x - \frac{x^3}{30}\right]_0^{10} - 800 \\ \Rightarrow CS &= \left(900 - \frac{1000}{30}\right) - 800 = \frac{200}{3} = 66.667 \end{aligned}$$

Since x is in thousands, so the consumer's surplus is ₹ 66667

ii. The producer's surplus (PS) is given by

$$\begin{aligned} PS &= p_0 x_0 - \int_0^{x_0} S(x)dx \\ \Rightarrow PS &= 80 \times 10 - \int_0^{10} \left(\frac{1}{5}x^2 + x + 50\right) dx \\ \Rightarrow PS &= 800 - \left[\frac{x^3}{15} + \frac{x^2}{2} + 50x\right]_0^{10} \\ \Rightarrow PS &= 800 - \left(\frac{1000}{15} + \frac{100}{2} + 500\right) = 800 - \left(\frac{200}{3} + 50 + 500\right) = \frac{550}{3} = 183.33 \end{aligned}$$

So, the producer's surplus is ₹ 183333

29. We have, $\mu = 30$ and $\sigma = 5$

Let Z be the standard normal variate. Then,

$$Z = \frac{X - \mu}{\sigma} \Rightarrow Z = \frac{X - 30}{5}$$

i. When $X = 26$, we obtain: $Z = \frac{26 - 30}{5} = -0.8$

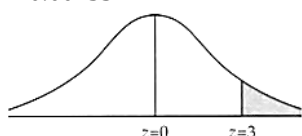
When $X = 40$, we obtain: $Z = \frac{40 - 30}{5} = 2$

$$\begin{aligned} \therefore P(26 \leq X \leq 40) &= P(-0.8 \leq Z \leq 2) \\ &= P(-0.8 \leq Z \leq 0) + P(0 \leq Z \leq 2) \\ &= P(0 \leq Z \leq 0.8) + P(0 \leq 2 \leq 2) \\ &= 0.2881 + 0.4772 = 0.7563 \text{ [See table]} \end{aligned}$$



ii. When $X = 45$, we obtain

$$\begin{aligned} Z &= \frac{45 - 30}{5} = 3 \\ \therefore P(X \geq 45) &= P(Z \geq 3) \\ &= 0.5 - P(0 \leq Z \leq 3) \\ &= 0.5 - 0.49865 \text{ [See Table]} \\ &= 0.00135 \end{aligned}$$



$$\begin{aligned} \text{iii. } P(|X - 30| > 5) &= 1 - P(|X - 30| \leq 5) \\ &= 1 - P(30 - 5 \leq X \leq 30 + 5) \end{aligned}$$

$$= 1 - P(25 \leq X \leq 35)$$

$$\text{Now, } X = 25 \Rightarrow Z = \frac{25-30}{5} = -1 \text{ and, } X = 35 \Rightarrow Z = \frac{35-30}{5} = 1.$$

$$\therefore P(|X - 30| > 5)$$

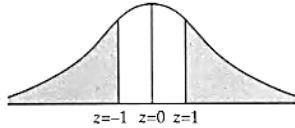
$$= 1 - P(-1 \leq Z \leq 1)$$

$$= 1 - 2 \cdot P(0 \leq Z \leq 1)$$

$$= 1 - 2 \times 0.3413$$

$$= 1 - 0.6826$$

$$= 0.3174$$



OR

Let X is the random variable that denotes that a bulb is defective.

Also, $n = 10$, $p = \frac{1}{50}$ and $q = \frac{49}{50}$ and $P(X = r) = {}^nC_r p^r q^{n-r}$

i. None of the bulbs are defective i.e., $r = 0$

$$\therefore P(X = r) = P_{(0)} = {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} = \left(\frac{49}{50}\right)^{10}$$

ii. Exactly two bulbs are defective i.e., $r = 2$

$$\begin{aligned} \therefore P(X = r) &= P_{(2)} = {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8 \\ &= \frac{10!}{8!2!} \left(\frac{1}{50}\right)^2 \cdot \left(\frac{49}{50}\right)^8 = 45 \times \left(\frac{1}{50}\right)^2 \times (49)^8 \end{aligned}$$

iii. More than 8 bulbs work properly i.e., there are less than 2 bulbs that are defective.

So, $r < 2 \Rightarrow r = 0, 1$

$$\begin{aligned} \therefore P(X = r) &= P(r < 2) = P(0) + P(1) \\ &= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^{10-1} \\ &= \left(\frac{49}{50}\right)^{10} + \frac{10!}{1!9!} \cdot \frac{1}{50} \cdot \left(\frac{49}{50}\right)^9 \\ &= \left(\frac{49}{50}\right)^{10} + \frac{1}{5} \cdot \left(\frac{49}{50}\right)^9 = \left(\frac{49}{50}\right)^9 \left(\frac{49}{50} + \frac{1}{5}\right) \\ &= \left(\frac{49}{50}\right)^9 \left(\frac{59}{50}\right) = \frac{59(49)^9}{(50)^{10}} \end{aligned}$$

30. Computation of three-yearly moving averages

Years	Profit (₹)	3-yearly moving Total (₹)	3-yearly moving average (₹)
2016	15,420	-	-
2017	15,470	46,410	15,470
2018	15,520	52,010	17,336.667
2019	21,020	63,040	21,013.333
2020	26,500	79,470	26,490
2021	31,950	94,050	31,350
2022	35,600	94,050	31,350
2023	34,900	-	-

The last column gives the trend of profits.

31. We have,

Population mean (μ) = 140, Sample mean ($\bar{x}$) = 147

Sample size (n) = 26, Standard deviation (s) = 16

Now, we define

Null Hypothesis H_0 : There is no significant difference in the mean sales before and after advertisement.

Alternate Hypothesis H_1 : There is significant difference in the mean sales before and after advertisement. Let t be the test statistic given by

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \text{ or, } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$\Rightarrow t = \frac{147-140}{\frac{16}{\sqrt{26-1}}} \text{ or, } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

The sample statistic 't' follows students t distribution with $v = (26 - 1) = 25$ degrees of freedom. Now, It is given that $t_{25}(0.05) = 2.06$.
 $\therefore |t| = 2.187 > 2.06 = t_{25}(0.05)$

i.e., Calculated $|t| < \text{tabulated } t_{25}(0.05)$

Therefore, we reject the null hypothesis and accept the alternate hypothesis H_1 .

Hence, we conclude that advertisement is effective for sales.

Section D

32. Let x and y be the number of packages of screws of type A and B respectively manufactured by the factory, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in ₹) $Z = 7x + 10y$ subject to the constraints

$4x + 6y \leq 240$ (automatic machine constraint)

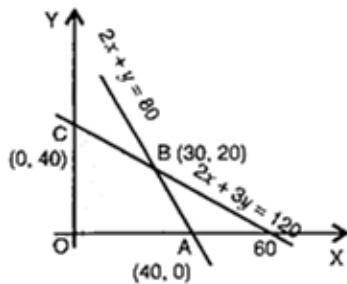
i.e. $2x + 3y \leq 120$

$6x + 3y \leq 240$ (hand-operated machine constraint)

i.e. $2x + y \leq 80$

$x \geq 0, y \geq 0$ (non-negativity constraints)

Draw the lines $2x + 3y = 120$ and $2x + y = 80$, and shade the region satisfied by the above inequalities. The feasible region is the polygon OABC, which is convex and bounded.



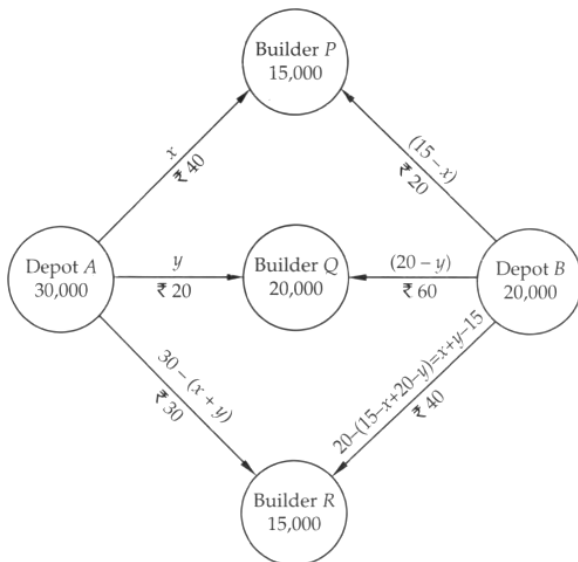
The corner points are $O(0, 0)$, $A(40, 0)$, $B(30, 20)$ and $C(0, 40)$.

The values of Z (in ₹) $= 7x + 10y$ at the points O, A, B and C are 0, 280, 410 and 400 respectively.

$\therefore$ Maximum profit = ₹ 410, when 30 packages of screws of type A and 20 packages of screws of type B are manufactured.

OR

The given information can be exhibited diagrammatically as shown in Fig.



Let depot A transport x thousands of bricks to builders P, y thousands to builder Q. Since depot A has a stock of 30,000 bricks. Therefore, the remaining bricks i.e. $30 - (x + y)$ thousands of bricks will be transported to the builder R.

Since the number of bricks is always a non-negative real number. Therefore,

$x \geq 0, y \geq 0$ and $30 - (x + y) \geq 0 \Rightarrow x \geq 0, y \geq 0$ and $x + y \leq 30$

Now, the requirement of the builder P is of 15000 bricks and x thousand bricks are transported from depot A. Therefore, the

remaining $(15 - x)$ thousands of bricks are to be transported from the depot at B. The requirement of the builder Q is of 20,000 bricks and y thousand bricks are transported from depot A. Therefore, the remaining $(20 - y)$ thousand bricks are to be transported from depot B.

Now, depot B has $20 - (15 - x + 20 - y) = x + y - 15$ thousand bricks which are to be transported to the builder R.

Also, $15 - x \geq 0$, $20 - y \geq 0$ and $x + y - 15 \geq 0 \Rightarrow x \leq 15$, $y \leq 20$ and $x + y \geq 15$

The transportation cost from the depot A to the builders P, Q and R are respectively ₹ 40x, 20y and 30 $(30 - x - y)$. Similarly, the transportation cost from the depot B to the builders P, Q and R are respectively ₹ 20 $(15 - x)$, 60 $(20 - y)$ and 40 $(x + y - 15)$ respectively. Therefore, the total transportation cost Z is given by

$$Z = 40x + 20y + 30(30 - x - y) + 20(15 - x) + 60(20 - y) + 40(x + y - 15)$$

$$\Rightarrow Z = 30x - 30y + 1800$$

Hence, the above LPP can be stated mathematically as follows:

Find x and y in thousands which

$$\text{Minimize } Z = 30x - 30y + 1800$$

Subject to

$$x + y \leq 30$$

$$x \leq 15$$

$$y \leq 20$$

$$x + y \geq 15$$

$$\text{and, } x \geq 0, y \geq 0$$

33. We find that $7^1 \equiv 7 \pmod{13}$

$$7^2 \equiv 7 \cdot 7 = 49 \pmod{13}$$

$$7^2 \equiv 10 \pmod{13} [49 = 10 \pmod{13}]$$

$$7^3 \equiv 70 \pmod{13}$$

$$7^3 \equiv 5 \pmod{13} [70 \equiv 5 \pmod{13}]$$

$$7^4 \equiv 35 \pmod{13}$$

$$7^4 \equiv 9 \pmod{13} [35 \equiv 9 \pmod{13}]$$

$$7^5 \equiv 63 \pmod{13}$$

$$7^5 \equiv -2 \pmod{13} [63 \equiv -2 \pmod{13}]$$

$$7^6 \equiv -14 \pmod{13}$$

$$7^6 \equiv 1 \pmod{13} [-14 \equiv -1 \pmod{13}]$$

$$7^{258} = (7^6)^{43}$$

$$7^6 \equiv -1 \pmod{13}$$

$$(7^6)^{43} \equiv (-1)^{43} \pmod{13}$$

$$7^{258} \equiv -1 \pmod{13}$$

$$7^{258} \equiv 12 \pmod{13} [-1 \equiv 12 \pmod{13}]$$

34. The average number of errors per page = $\frac{4}{10} = 0.4$

Thus, if m is the mean of the Poisson's distribution, then $m = 0.4$

Let X be the random variable denoting the number of errors per page

$$\text{Then, } P(X = r) = \frac{m^r e^{-m}}{r!} = \frac{(0.4)^r e^{-0.4}}{r!} = 0.6703 \times \frac{(0.4)^r}{r!} \dots (i)$$

Let $f(r)$ denote the number of pages, each containing r errors, in 1000 pages

Then,

$$f(r) = 1000 \times P(X = r)$$

$$= 1000 \times 0.6703 \times \frac{(0.4)^r}{r!} \text{ [Using (i)]}$$

$$= 670.3 \times \frac{(0.4)^r}{r!} \dots (i)$$

Putting $r = 0, 1, 2, 3$ and 4 in (ii), we get

$$f(0) = 670.3 \times \frac{(0.4)^0}{0!} = 670.3$$

$$f(1) = 670.3 \times \frac{(0.4)^1}{1!} = 268.12$$

$$f(2) = 670.3 \times \frac{(0.4)^2}{2!} = 53.624$$

$$f(3) = 670.3 \times \frac{(0.4)^3}{2!} = 7.1498$$

$$f(4) = 670.3 \times \frac{(0.4)^4}{4!} = 0.71498$$

Hence, the number of pages containing 0, 1, 2, 3 and 4 errors are 670, 268, 54, 7 and 1 respectively.

OR

Given the average event of flood overflow in every 10 years is 2.

So, $\lambda = 2$.

$\therefore$ Required probability = $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!}$$

$$= e^{-2} \left(1 + 2 + 2 + \frac{4}{3} \right)$$

$$= 0.135 \times \frac{19}{3} = 0.045 \times 19$$

$$= 0.855 = 0.86$$

35. CAGR is the mean annual growth rate of an investment over a specified period of time longer than one year.

$$\text{CAGR} = \left[\frac{\text{Ending investment amount}}{\text{Start amount}} \right]^{\frac{1}{\text{no. of years}}} - 1$$

P.V. = ₹ 10,000

F.V. = ₹ 14,000

n = 6 years

$$\text{So, CAGR} = \left(\frac{14000}{10000} \right)^{\frac{1}{6}} - 1 = (1.4)^{\frac{1}{6}} - 1$$

$$= 1.058 - 1$$

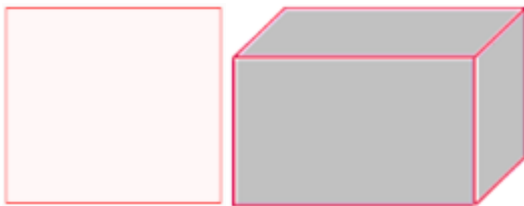
$$= 0.058$$

Hence, CAGR = 5.8%

Section E

36. Read the text carefully and answer the questions:

Yash wants to prepare a handmade gift box for his friend's birthday at his home. For making lower part of the box, he took a square piece of paper of each side equal to 10 cm.



(i) (0,10)

(ii) $V = x(10 - 2x)(10 - 2x)$

(iii) $\frac{5}{3}, 5$

OR

$$\frac{5}{3} \text{ cm}$$

37. Read the text carefully and answer the questions:

Understanding Perpetuity

An annuity is a stream of cash flows. A perpetuity is a type of annuity that lasts forever, into perpetuity. The stream of cash flows continues for an infinite amount of time. In finance, a person uses the perpetuity calculation in valuation methodologies to find the present value of a company's cash flows when discounted back at a certain rate.

An example of a financial instrument with perpetual cash flows was the British-issued bonds known as consols, which the Bank of England phased out in 2015. By purchasing a consol from the British government, the bondholder was entitled to receive annual interest payments forever.

Perpetuity Present Value Formula

The formula to calculate the present value of perpetuity or security with perpetual cash flows is as follows:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} \dots = \frac{C}{r}$$

where:

PV present value

C = cash flow

r = discount rate

(i) ₹ 18000

(ii) ₹ 25000

(iii) ₹ 10300

OR

₹ 2880

$$38. A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad A' = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A')$$
$$= \frac{1}{2} \left(\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 14 & -4 & -4 \\ -4 & 2 & 0 \\ -4 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = P$$

∴ P is symmetric Matrix.

$$Q = \frac{1}{2}(A - A')$$
$$= \frac{1}{2} \left(\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} = -Q$$

⇒ Q is skew symmetric.

P + Q

$$= \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

= A.

i.e. P + Q = A

OR

The total sales (S) in thousands of rupees for products X and Y is given by

$$S = a + bX + cY = 12 \dots (i)$$

In January, we have

$$X = 2, Y = 3 \text{ and } S = 12$$

Substituting these values in $S = a + bX + cY$, we obtain

$$a + 2b + 3c = 12$$

Similarly, for February and March months, we obtain

$$a + 6b + 2c = 13 \text{ [Putting } X = 6, Y = 2, S = 13 \text{ in (i)]}$$

$$a + 5b + 3c = 15 \text{ [Putting } X = 5, Y = 3, S = 15 \text{ in (i)]}$$

Thus, we obtain the following system of equations

$$a + 2b + 3c = 12$$

$$a + 6b + 2c = 13$$

$$a + 5b + 3c = 15$$

$$\therefore D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 6 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 1(18 - 10) - 2(3 - 2) + 3(5 - 6) = 3$$

$$D_1 = \begin{vmatrix} 12 & 2 & 3 \\ 13 & 6 & 2 \\ 15 & 5 & 3 \end{vmatrix} = 12(18 - 10) - 2(39 - 30) + 3(65 - 90) = 3$$

$$D_2 = \begin{vmatrix} 1 & 12 & 3 \\ 1 & 13 & 2 \\ 1 & 15 & 3 \end{vmatrix} = 1(39 - 30) - 12(3 - 2) + 3(15 - 13) = 3$$

$$\text{and, } D_3 = \begin{vmatrix} 1 & 2 & 12 \\ 1 & 6 & 13 \\ 1 & 5 & 15 \end{vmatrix} = 1(90 - 65) - 2(15 - 13) + 12(5 - 6) = 9$$

Using Cramer's rule, the solution is given by

$$a = \frac{D_1}{D} = \frac{3}{3} = 1, b = \frac{D_2}{D} = \frac{3}{3} = 1, \quad c = \frac{D_3}{D} = \frac{9}{3} = 3$$

Substituting the values of a, b and c in $S = a + bX + cY$, we obtain

$$S = 1 + X + 3Y$$

When $X = 4$ and $Y = 5$, we obtain

$$S = 1 + 4 + 15 = 20$$

Hence, the sales in the month of April is of ₹ 20,000.